

Fringing field effects on first-order focusing in magnetic spectrographs

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With 4 figures in the text

Abstract

The fringing flux influence on ion paths in the median plane of magnetic spectrographs of the short uniform-field type is investigated, and the corrections to be applied to the ideal-field theory are given to the first order of approximation. This influence consists in displacing the posterior principal point of the ion lens connected with the deflecting field without altering the focal distance. In double-focusing spectrographs the conditions of simultaneous velocity and directional focusing will be modified.

1. Introduction

In connection with magnetic spectrograph work, the ion optics of the fringing field of deflecting magnets have received considerable attention (*e.g.*, [1–3]). Some theoretical and experimental investigations [4, 5] deal exclusively with the fringing field influence on the position of the spectral focus of the analyzer. In these investigations the deflecting fields considered have been of the uniform sector-type fields. The ion optics of these have been worked out in great detail in the ideal case of no fringing field [6, 7]. In a thorough investigation into the ion optics of an inhomogeneous sector-type electric-magnetic field [8] the effects of stray fields have been considered briefly.

These results however only apply to the case of ion paths traversing the field boundary at (nearly) right angles. It is the purpose of the following investigation to extend these calculations to include the more general case of oblique ion paths at the field boundary. This should be of some interest, as theory [9, 10] indicates the possibility of obtaining second-order focusing in such case with straight lines as field boundaries. Generally speaking, the theory of second-order focusing in magnetic spectrographs of the short field type must consider the fringing field effects [5]. The present work however will be limited to the first-order effects.

2. The fringing field

The field strength distribution in the median plane of a magnet with parallel pole-faces has been calculated for several pole-piece configurations [4, 11] by

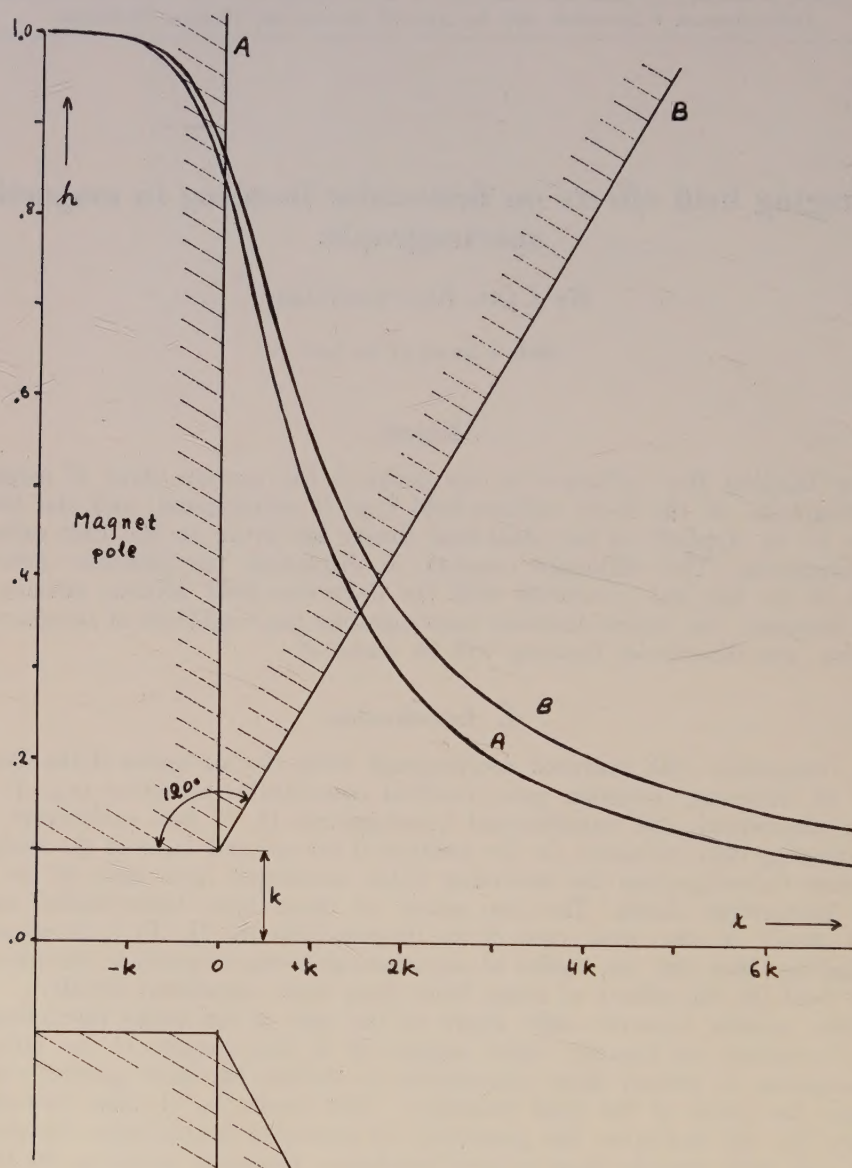


Fig. 1. Configuration of magnet poles and corresponding field shape.

considering the pole surface as constituting an equipotential surface. Measurements [5] seem to corroborate the validity of this method even at fairly high flux densities. The calculations refer usually to poles with infinite extension. In practice the limited pole height and breadth will introduce differences which may not be small. The individual field of the magnet must then be measured

in a large region of the median plane, but the results may be treated in the same manner as the particular field distributions considered below.

The following calculations refer to two cases of infinite, planeparallel pole-pieces with plane lateral faces making an edge angle of 90° or 120° , as displayed in Fig. 1. The analytical expressions were obtained by applying the ordinary methods of conformal mapping by analytic transformations of the positional coordinates (the case of 90° being treated already by J. J. THOMSON [12]). They read:

$$90^\circ: \pi x = \frac{2}{h} - \ln \frac{1+h}{1-h} \quad (1A)$$

$$120^\circ: \pi x = \frac{3}{h} + \frac{1}{2} \ln \frac{(1-h)^3}{1-h^3} - \sqrt{3} \arctan \frac{1+2h}{\sqrt{3}}. \quad (1B)$$

Here the unit of field strength is the field strength in the inner region of the magnetic gap, which is not influenced by the boundary perturbations. h denotes the field strength in the median plane at the distance x from the projection of the pole-face boundary. One half of the gap width, $2k$, is taken as unit of length.

It will be seen from the diagram, that h rapidly approaches 1 when x decreases from the origin to negative values. In the other direction, h decreases slowly and is asymptotically proportional to x^{-1} . Because of this slow decrease, it is not permissible to consider the fringing field as limited to the vicinity of the gap. Rather, its extension is determined by the dimensions of the pole-pieces. At a distance larger than the smallest of these dimensions, the field will fall off at a higher rate than that given by eqs. (1).

3. Ideal field optics

Consider the case of an ideal uniform magnetic sector field (Fig. 2) deflecting an ion beam in the median plane, the geometry being such, that the ion beam intersects the field boundary at right angles. The field acts as a (cylindrical) lens with the principal points coinciding at the intersection of the extensions of the straight parts of the beam. The focal distance is [6]:

$$f_\phi = a \cdot \cot \frac{1}{2} \Phi. \quad (2)$$

If one field boundary is rotated so as to make an angle ε with the ion beam, then, ion optically, a thin lens will be introduced of focal distance

$$f_\varepsilon = -a \cot \varepsilon. \quad (3)$$

We will now find an analytic series expression for a principal ray in the vicinity of the field boundary. For this purpose, the rectangular coordinate system (x, y) of Fig. 3 is placed with the field boundary parallel to the y axis at $x = \xi$. Our principal ray should then pass through $(\xi, 0)$. If $\varphi(x)$ denotes the angle of direction of the tangent to the ion path at any point (x, y) , then:

$$a d\varphi = -h \cdot \sec \varphi \cdot dx.$$

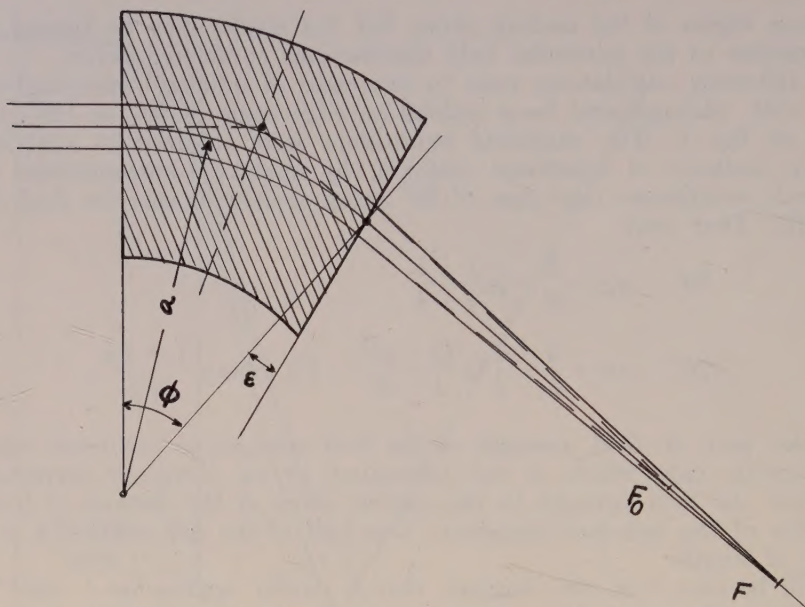


Fig. 2. Focusing by a short uniform field.

F_0 : focus of field with $\varepsilon = 0$.

From this is derived the following general equation of path:

$$y' = \tan \varphi = -a^{-1} \cdot \int \sec^3 \varphi \cdot h \, dx. \quad (4)$$

In the present case of an ideal field, $h = 1$ inside the boundary $x = \xi$, and $h = 0$ outside of the same boundary. In the latter region, the path reduces to a straight line of angle $-\varepsilon = \arctan y'$. Otherwise, the integrand of (4) may be developed into a power series of $(x - \xi)$. The resulting expressions are:

$$y'_i = -\tan \varepsilon - h_i \cdot a^{-1} \cdot \sec^3 \varepsilon \cdot (x - \xi) + \dots \quad (5)$$

$$y_i = -(x - \xi) \tan \varepsilon - h_i \cdot a^{-1} \cdot \frac{1}{2} \sec^3 \varepsilon \cdot (x - \xi)^2 + \dots \quad (6)$$

Subscript "i" indicates variables of the ideal field path.

4. Ion path displacement due to stray field

We consider now the ion path in the real field, corresponding to the principal ray of eqs. (5) and (6). The difference of the two path functions is by (4):

$$\delta y' = y' - y'_i = -a^{-1} \cdot \sec^3 \varepsilon \cdot \int (h - h_i) \, dx + \dots \quad (7)$$

$$\delta y = y - y_i = -a^{-1} \cdot \sec^3 \varepsilon \cdot \iint (h - h_i) \, dx^2 + \dots \quad (8)$$

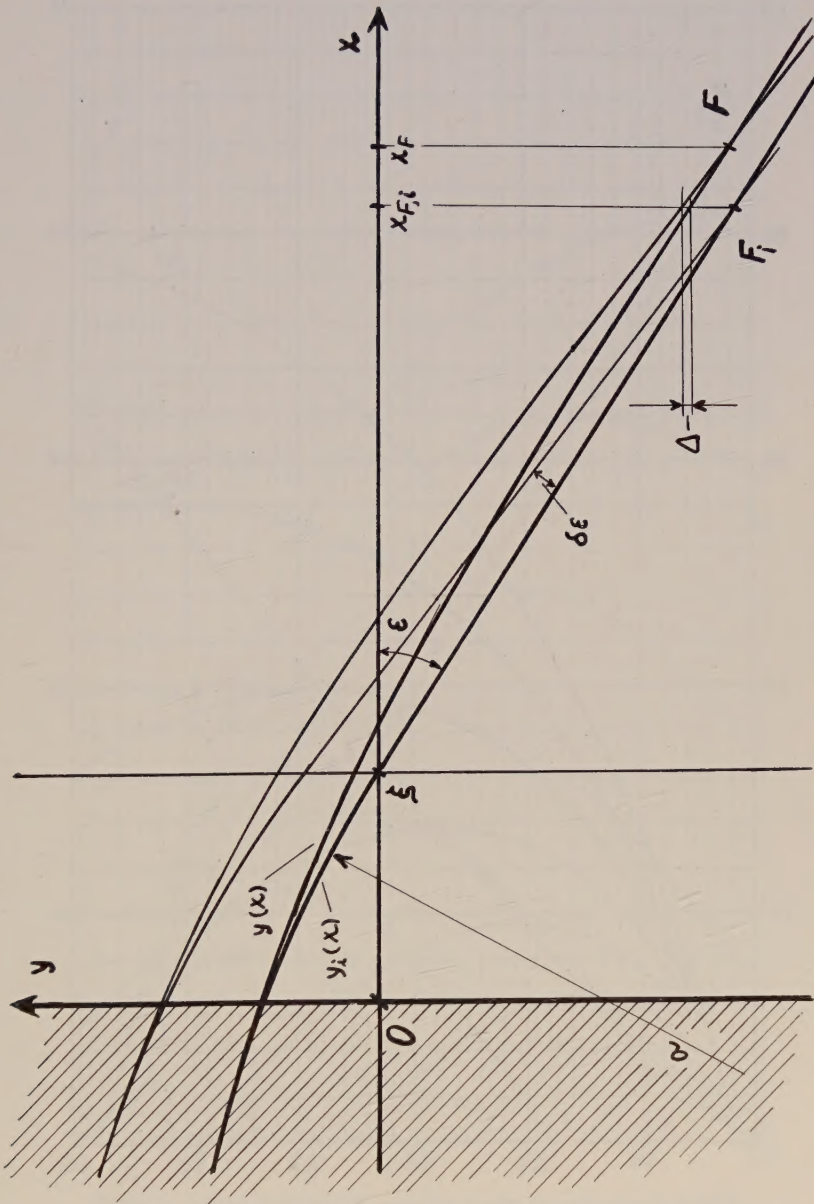


Fig. 3. Ion paths in ideal and real field.

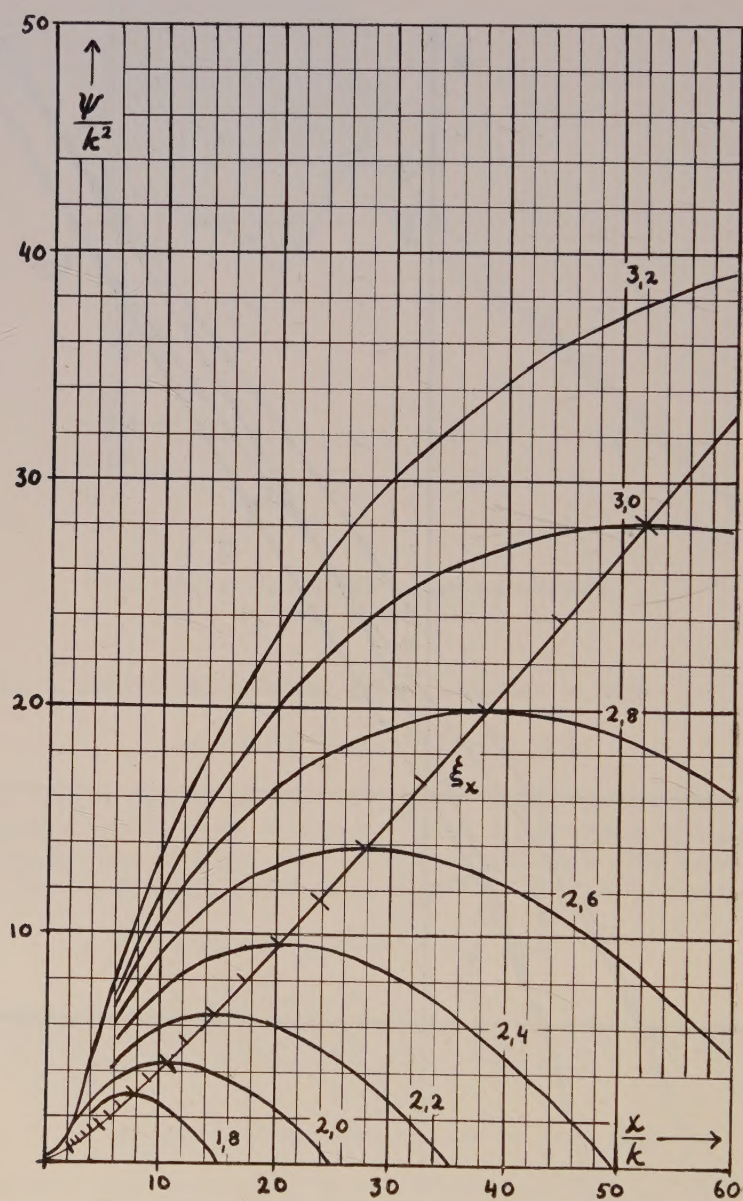


Fig. 4 a. Case A of fig. 1 — edge angle of pole-pieces 90° .

Parameters: ξ in units of k .

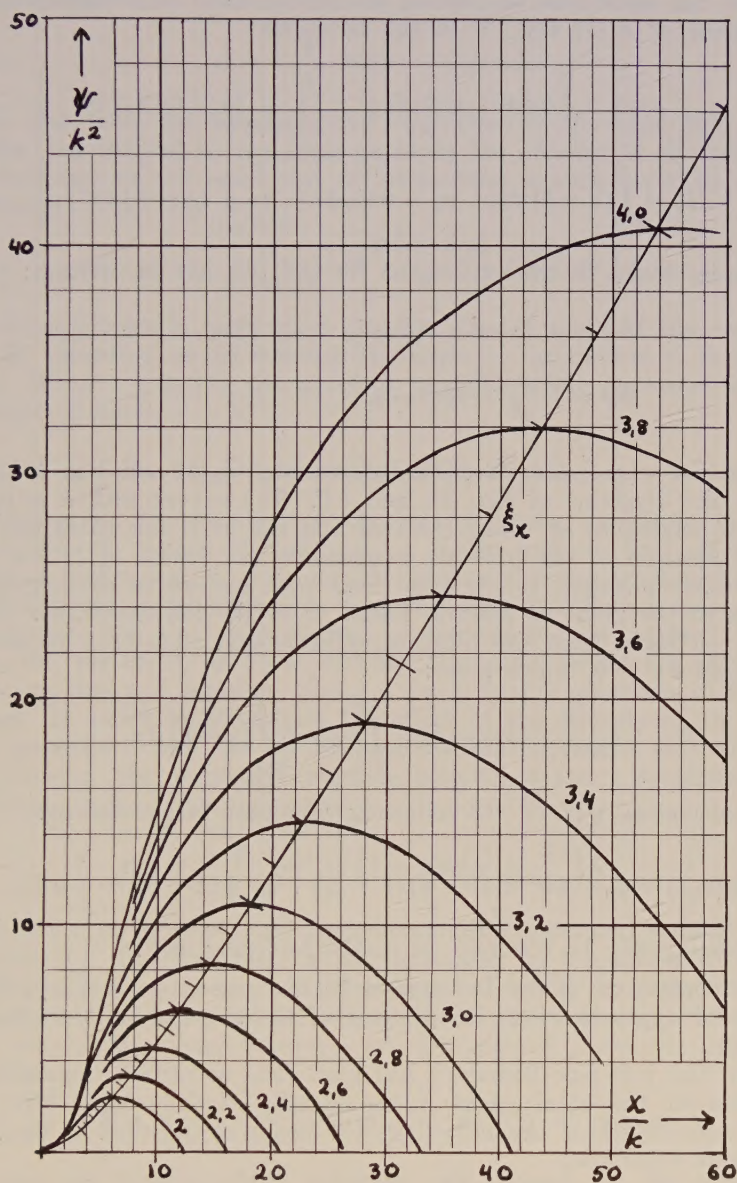


Fig. 4 b. Case B of fig. 1 — edge angle of pole-pieces 120° .

Parameters: ξ in units of k .

The paths should be identical in the inner uniform part of the field, where $h = 1$. We can state this by placing the lower limits of integration intervals in this region, at $x(1)$ say. With the notations:

$$\left. \begin{aligned} \kappa(x) &= \int_0^x h dx; & \psi(x, \xi) &= - \int_{x(1)}^x \int_{x(1)}^x (h - h_i) dx^2; \\ \xi_0 &= \int_{x(1)}^0 (1 - h) dx; & \psi_0 &= \psi(0, \xi) = \int_{x(1)}^0 \int_{x(1)}^x (1 - h) dx^2; \end{aligned} \right\} \quad (9)$$

the first-order terms of the expansions (7) and (8) may be written:

$$\delta y' = a^{-1} \cdot \sec^3 \varepsilon \cdot (\xi + \xi_0 - \kappa), \text{ when } x > \xi \quad (10)$$

$$\delta y = \frac{\psi(x, \xi)}{a^2} \cdot \sec^3 \varepsilon \cdot a. \quad (11)$$

The function $\psi(x, \xi)$ for the field distributions of (1) and Fig. 1 have been plotted in the diagrams of Fig. 4 a and 4 b. The curves with ξ as parameter all show a maximum, at which the real and the ideal ion paths are parallel. By proper location of the pole-face boundaries, i.e.: choice of ξ , the real field may be made equivalent to the ideal field with respect to the angular deflection Φ up to the point of spectral focus, or to the ion source slit. This value of ξ may be taken from the diagram or calculated from (10) by introducing the condition $\delta y' = 0$, which gives:

$$\xi_{x_F} = \kappa(x_F) - \xi_0 = \int_0^{x_F} h dx - \int_{x(1)}^0 (1 - h) dx. \quad (12)$$

When calculating $\psi(x, \xi)$, the following expansion is convenient:

$$\psi(x, \xi) = \psi_0 + \frac{1}{2} \xi^2 + (x - \xi)(\xi + \xi_0) + h_i \cdot \frac{1}{2} (x - \xi)^2 - \int_0^x \kappa dx. \quad (13)$$

These results may be summed up in the following way:

1) The correction, ξ , to be applied to the pole-face boundary, is (to the first order of approximation) independent of the angle of ion beam inclination, ε (eq. (12)).

2) When the pole-face boundary has been thus corrected, the real ion paths are parallel to the ideal paths in the vicinity of the image. This signifies, that the focal length of the deflecting-field lens is not affected by the boundary perturbations considered.

5. The displacement of the ion optical image

At the image point of the ideal-field optic axis, $(x_{F,i}, y_{F,i})$, where $y_{F,i} = -(x_{F,i} - \xi) \tan \varepsilon$, let a second ray of different angle ($\varepsilon \rightarrow \varepsilon + \delta \varepsilon$) intersect this axis. The corresponding rays of the real field have here a separation Δ

parallel to the y axis, which is given by the differential of the path function (11) with respect to ε . Noting that $x_{F,i} > \xi$ and accordingly $h_i = 0$, we get:

$$\Delta = 3 \cdot \frac{\psi}{a^2} \cdot \sin \varepsilon \cdot \sec^4 \varepsilon \cdot a \delta \varepsilon. \quad (14)$$

If the real field has been rendered equivalent to the ideal field, then the real paths are parallel to the ideal paths in the vicinity of the image point. The x -coordinate of the real point of intersection is then found by dividing Δ with $d(\tan \varepsilon)$ — see Fig. 3:

$$x_F - x_{F,i} = \frac{\Delta}{d(\tan \varepsilon)} = 3 \frac{\psi(x_F, \xi_{x_F})}{a^2} \cdot \sin \varepsilon \cdot \sec^3 \varepsilon \cdot a. \quad (15)$$

This displacement is positive or negative (in the sense of the coordinate x of Fig. 3) according as to whether the angle of inclination, ε , is positive or negative. When $\varepsilon = 0$, the displacement is zero (to the degree of approximation considered here).

6. Stray field effects on double-focusing

The transverse displacement of the ion paths is, as shown by eq. (11), dependent on the radius of curvature, a , in the uniform field, as well as on the angle of inclination, ε . In a double-focusing mass spectrograph¹ there exist paths of different a values, which leave the field at different inclinations but — according to the ideal-field theory — converge on the spectral focus at x_F . Now, in the real field, as δy is varying, this is no longer the case, and the mass-spectral line will broaden, giving a small energy spectrum of the ions of each mass. In order to investigate the effect of this broadening on the mass-spectral resolution, one has to differentiate the expression (11) with respect to a , observing the particular relation between a and ε determined by the beam-defining slits of the apparatus:

$$\delta^2 y = - \frac{\psi}{a^2} \cdot \sec^3 \varepsilon \cdot \left(1 - 3 \sin \varepsilon \cdot a \cdot \frac{d\varepsilon}{da} \right) \delta a. \quad (14)$$

We consider at present only instruments with separated electric and magnetic fields [6]. In this case, it will generally be impossible to reduce the parenthesis of (14) to zero. It will however be possible to restore the double-focusing by a small experimental adjustment of the field geometry, which will give an ideal-field energy-dispersion at the spectral focus just compensating the dispersion (14):

$$\delta y_i = - \delta^2 y = \frac{\psi}{a^2} \cdot \sec^3 \varepsilon \cdot \left(1 - 3 \sin \varepsilon \cdot a \cdot \frac{d\varepsilon}{da} \right) \delta a. \quad (15)$$

The manner of achieving this condition will depend on the constructional means of the particular spectrographs. *E.g.*, a rotation of one field in respect to the other may be found adequate and convenient.

¹ In recent literature (*e.g.*, (13)) the word *double-focusing* has been used to indicate re-focusing of spatially diverging rays. It should however be reserved to the simultaneous angular and velocity focusing embodied since 1934 in the instruments of DEMPSTER, MATTAUCH & HERZOG, and others.

7. Conclusion and discussion

The general first-order effect of the fringing field of a properly corrected magnet is, then, *only* that of displacing the ion optical image. The ion optical *thin* lens of focal distance f_e , which represents the dioptric constants of the ideal field boundary, is in the real field replaced by a *thick* lens of the same focal distance.

In contrast to the method adopted here, PLOCH & WALCHER [5] have calculated the image displacement in case the pole faces are congruent to the sector fields of the ideal-field theory. Instead they pay due consideration to the field distribution in the XY plane outside the magnet. In common configurations this may introduce an angle between the pole-face boundary and the effective boundary (which in the present theory is defined by ξ and made to coincide with the ideal field boundary). Disregarding this effect, the difference between the two methods may be stated as follows: P. & W. find in the case $\varepsilon = 0$ the image distance from the magnet unaltered as compared to the ideal-field theory, but the deflection Φ and the inclination ε are changed by approximately ξ/a . The present treatment gives the image distance from the effective field boundary unaltered with the deflection and inclination likewise unchanged. In fact, according to eq. (45'') of ref. [6], the tangential image displacement due to a small parallel displacement of the posterior field boundary, is zero, when $\varepsilon \approx 0$.

On the other hand, COGGESHALL [4] arrived at quite a different result, *viz.*, that (in the case of $\varepsilon = 0$) the pole-image distance *and* the deflection angle should be left unaltered by the fringing field. The method of reasoning adopted was however not exact.

From the experimental point of view, the fringing field effects usually are small, and it is a difficult task to corroborate the theory quantitatively by experiment. In such control experiments the distribution of the flux in the XY plane should be established by measurements in order to enable the calculation of ξ and ε . Constructional imperfections or irregularities of the magnet pole-faces will easily mask the fringing field effects (*cf.*, the work of PLOCH & WALCHER).

Fysiska Institutionen, Uppsala universitet, December 1950.

REFERENCES. [1] Dempster, 1918, Phys. Rev. *11*, 316. — [2] Smythe, Rumbaugh & West, 1934, Phys. Rev. *45*, 724. — [3] Bainbridge & Jordan, 1936, Phys. Rev. *50*, 282. — [4] Coggeshall, 1947, J. Appl. Phys. *18*, 855. — [5] Ploch & Walcher, 1950, Z. Physik *127*, 274. — [6] Herzog, 1934, Z. Physik *89*, 447. — [7] Hutter, 1945, Phys. Rev. *67*, 248. — [8] Marschall, 1944, Phys. Z. *45*, 1. — [9] Kerwin, 1949, Rev. Sc. Instr. *20*, 36. — [10] Hintenberger, 1948, Rev. Sc. Instr. *20*, 748. — [11] Rose, 1938, Phys. Rev. *53*, 715. — [12] Thomson, Recent Researches in Electricity and Magnetism (Oxford, 1893). — [13] Rosenblum, 1950, Rev. Sc. Instr. *21*, 586.

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(Arkiv för Fysik. Band 3 nr 5, p. 53–62.)

Errata

P. 53, l. 2 (caption); for 1951, read: 1950.

P. 55, formula (2); read:

$$f_{\Phi} = a \cdot \operatorname{cosec} \Phi. \quad (2)$$

P. 61, l. 8; read:

$$x_F - x_{F,i} = \frac{\Delta}{d (\tan \varepsilon)} = 3 \frac{\psi(x_F, \xi_{x_F})}{a^2} \cdot \sin \varepsilon \cdot \sec^2 \varepsilon \cdot a. \quad (15)$$

P. 61, l. 25; read:

$$\delta^2 y = -\frac{\psi}{a^2} \cdot \sec^3 \varepsilon \cdot \left(1 - 3 \operatorname{tg} \varepsilon \cdot a \cdot \frac{d\varepsilon}{da}\right) \delta a. \quad (16)$$

P. 61, l. 28 and 31; for (14), read: (16).

P. 61, l. 32; read

$$\delta y_i = -\delta^2 y = \frac{\psi}{a^2} \cdot \sec^3 \varepsilon \cdot \left(1 - 3 \operatorname{tg} \varepsilon \cdot a \cdot \frac{d\varepsilon}{da}\right) \delta a. \quad (17)$$

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